

n	C	A	D	D	T_{\min}	T_{\max}	ST_{\min}	E_{\min}	ST_{avg}	ST_{\max}	U	R
1	1	1	1	0,0,1	1	1	1	1	1	1	1	1
2	2	2	2	0,1,1	2	2	2	2	2	2	2	2
3	5	12	6	1,1,1	4	8	3	3	4.667	6	4	4.5
4	14	55	18	1,1,2	11	30	7	7	11.510	18	8.666667	10.6
5	42	273	60	1,2,2	32	150	14	14	29.313	60	19.333333	25.6
6	132	1428	222	2,2,2	96	780	27	27	75.799	222	44.200000	62.828571
7	429	7752	794	2,2,3	305	4550	58	58	197.617	794	102.733333	155.958929
8	1430	43263	2988	2,3,3			127	127	517.247	2988	241.834921	390.444048
9	4862	246675	11856	3,3,3				266			574.914286	983.978857
10	16796	1430715	45580	3,3,4							1377.587302	2492.993468
11	58786	8414640	180960	3,4,4							3322.336508	6343.812317
12	208012	50067108	743160	4,4,4							8055.810467	16201.746633
limit	4.0	6.75	4.5								$\approx 2.48 \dots$	$\geq \approx 2.65 \dots$

n inner points in a triangular convex hull

C = Catalan numbers

A = abstract stacked triangulations = ternary trees with n inner nodes and $2n - 1$ leaves = $\binom{3n+1}{n}/(3n+1) \approx 6.75^n$

D = 3 chains, of lengths i, j, k . The limit exponent of 4.5 has been established by Marc.

T_{\min}, T_{\max} all triangulations of a point set (Oswin, from the database)

ST_{\min}, ST_{\max} stacked triangulations of a point set (Oswin, from the database)

ST_{avg} Average over all realizable order types of the given cardinality (Oswin)

E_{\min} lower bounds on stacked triangulations (Günter) It seems that the lower-bound examples (for n a multiple of 3) look like a series of nested concentric triangles, where successive levels are rotated by 180 degrees. The points lying close to a line through the center (they lie on both sides of the center) are probably not uniformly "curved" but they lie in such a way that a line through two such points cuts the points between them evenly. It remains to define such a family precisely and count the stacked triangulations for this family.

U = something random defined by the recursion $U_n = \sum_{i=1}^n U_{n-i} \cdot \frac{\sum_{j=1}^{i-1} U_{j-1} U_{i-1-j}}{i-1}$. (For $i = 1$, the value of the fraction is taken as 1.) Maybe this is something related to the degree-3-vertices?

R = average number of stacked triangulations on a random set, according to Emo's recursion $R_n = \sum_{i+j+k=n-1} R_i R_j R_k \cdot \frac{2}{n+1}$.

Prob[the balanced stacked triangulation with n inner vertices can be embedded on a random point set] $\approx 0.61886974^n$ (when n is of the form $(3^k - 1)/2$).

This is probably $>$ Prob[any other fixed stacked triangulation with n inner vertices can be embedded on a random point set].